AYRIM LI ALGEBRALARIDA LOKAL DIFFERENSIALLASHLAR VA LOKAL ANTI-DIFFERENSIALLASHLAR TAVSIFI

Arzikulov F.N.[1], Mutalov M. M.[2]

[1]*f.-m.f.d., O‘zR FA Matematika instituti*
[2]*Namangan davlat universiteti*
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ANNOTATSIYA

Maqolada haqiqiy sonlar maydoni ustida qurilgan ikki va uch o‘lchovli Abel bo‘lmagan Li algebralarining differensiallashlari, lokal differensiallashlari, anti-differensiallashlari va lokal anti-differensiallashlari fazolari o‘rganilib, to‘liq tavsiflangan. Mazkur operatorlar fazolarining algebraik tuzilishi aniqlanib, ular orasidagi o‘zaro munosabatlar ko‘rsatilgan. Olingan natijalar asosida ayrim Li algebralarida har qanday lokal differensiallash - differensiallash ekanligi, qolganlarida esa differensiallash bo‘lmagan lokal differensiallashlar mavjudligi isbotlangan. Shuningdek, ko‘rib chiqilgan barcha Li algebralarida anti-differensiallash bo‘lmagan lokal anti-differensiallashlar mavjudligi aniqlangan.

KALIT SO‘ZLAR

Li algebrasi, differensiallash, lokal differensiallash, anti-differensiallash, lokal anti-differensiallash, chiziqli akslantirish, chekli o‘lchovli algebra, avtomorfizm.

ОПИСАНИЕ ЛОКАЛЬНЫХ ДИФФЕРЕНЦИАЛИЗАЦИЙ И ЛОКАЛЬНЫХ АНТИДИФФЕРЕНЦИАЛИЗАЦИЙ В НЕКОТОРЫХ АЛГЕБРАХ LI

Арзикулов Ф.Н.[1], Муталов М.М.[2]

[1]*к.мат.н., Институт математики, Академия наук Республики Узбекистан*
[2]*Наманганский государственный университет*

АННОТАЦИЯ

В данной работе изучены и полностью описаны пространства дифференцирований, локальных дифференцирований, антидифференцирований и локальных антидифференцирований двумерных и трёхмерных неабелевых алгебр Ли над полем действительных чисел. Определена алгебраическая структура этих пространств операторов и установлены взаимосвязи между ними. На основе полученных результатов доказано, что в некоторых алгебрах Ли каждое локальное дифференцирование является дифференцированием, тогда как в остальных существуют локальные дифференцирования, не являющиеся дифференцированиями. Кроме того, установлено, что во всех рассмотренных алгебрах Ли существуют локальные антидифференцирования, не являющиеся антидифференцированиями.

КЛЮЧЕВЫЕ СЛОВА

алгебра Ли, дифференцирование, локальное дифференцирование, антидифференцирование, локальное антидифференцирование, линейное отображение, конечномерная алгебра, автоморфизм.

DESCRIPTION OF LOCAL DIFFERENTIALIZATIONS AND LOCAL ANTI-DIFFERENTIALIZATIONS IN CERTAIN LI ALGEBRAS

Arzikulov F.N.[1], Mutalov M. M.[2]

[1]*PhD in Mathematics, Institute of Mathematics, Academy of Sciences of the Republic of Uzbekistan*
[2]*Namangan State University*

ABSTRACT

The article studies and conducts overall characterization of the spaces of derivations, local derivations, anti-derivations, and local anti-derivations of two- and three-dimensional non-Abelian Lie algebras over the field of real numbers. The algebraic structure of these operator spaces is determined, and the relationships between them are established. Based on the obtained results, it is proved that in some Lie algebras every local derivation is a derivation, whereas in the others there exist local derivations that are not derivations. It is also established that local anti-derivations that are not anti-derivations exist in all of the considered Lie algebras.

KEYWORDS

Lie algebra, derivation, local derivation, anti-derivation, local anti-derivation, linear map, finite-dimensional algebra, automorphism.

Hozirgi zamon algebraik tuzilmalari va operatorlar nazariyasida Li algebralarini hamda ular ustidagi chiziqli operatorlar fazolarini o‘rganish eng muhim ilmiy yo‘nalishlardan biri sanaladi. Li algebralarining ichki xususiyatlari va geometrik qurilishini tadqiq etishda ushbu algebralardagi differensiallashlar fazosi asosiy algebraik invariant vazifasini bajaradi. Keyingi o‘n yilliklar mobaynida funksional analiz va kvant mexanikasi muammolari ta’siri ostida differensiallash tushunchasining turli umumlashgan ko‘rinishlarini, jumladan, lokal differensiallashlarni o‘rganishga bo‘lgan e’tibor sezilarli darajada kuchaydi.

Bundan tashqari, nazariy fizika va supersimmetrik maydonlar nazariyasi doirasida Li superalgebralari hamda ularga mos keluvchi operatorlar bilan bevosita bog‘liq tarzda anti-differensiallashlar tushunchasi shakllandi. Chekli o‘lchovli Li algebralarida differensiallashlar, lokal differensiallashlar, anti-differensiallashlar va lokal anti-differensiallashlar fazolarini ham alohida, ham o‘zaro aloqadorlikda batafsil tavsiflash (characterization) bu algebralarning avtomorfizmlari hamda kohomologiya guruhlarini topishda katta ahamiyatga ega. Mazkur operatorlar fazolarining strukturaviy qurilishini va ularning o‘lchovlarini belgilash masalasi tadqiqot mavzusining dolzarbligini ko‘rsatadi.

Ishning maqsadi chekli o‘lchovli Li algebralaridagi differensiallashlar, lokal differensiallashlar, anti-differensiallashlar va lokal anti-differensiallashlar fazolarining algebraik tuzilishini to‘liq tavsiflash hamda ular orasidagi o‘zaro munosabatlarni aniqlashdan iborat.

A bu algebra bo‘lsin. Agar chiziqli akslantirish $\delta: A \to A$ uchun, barcha $s,t \in A$ uchun $\delta(st) = s\delta(t) + \delta(s)t$ bo‘lsa, u holda δ differensiallash deb ataladi. Xuddi shunday, ixtiyoriy $s,t \in A$ uchun $\delta(st) = \delta(t)s + t\delta(s)$ bo‘lsa, δ anti-differensiallash deyiladi. Anti-differensiallashlar, masalan, [2] ishda uchburchaksimon halqalar misolida tadqiq etilgan. Bundan tashqari, agar chiziqli akslantirish $\nabla: A \to A$ va barcha $s \in A$ uchun shunday δ_s differensiallash topilsaki $\nabla(s) = \delta_s(s)$ bo‘lsa, u holda ∇ lokal differensiallash deb yuritiladi. O‘z navbatida, agar chiziqli akslantirish $\nabla: A \to A$ va barcha $s \in A$ uchun shunday δ_s anti-differensiallash topilsaki $\nabla(s) = \delta_s(s)$ bo‘lsa, u holda ∇ lokal anti-differensiallash deb ataladi.

Mazkur ishda haqiqiy sonlar maydoni ustida qurilgan 2 va 3 o‘lchovli Abel bo‘lmagan Li algebralarining [1] lokal differensiallashlari hamda lokal anti-differensiallashlari tavsiflab berilgan. Olingan natijalar quyida analitik shaklda keltirilgan.

1) $L(2;1)$: $e_1e_2 = e_1,\ e_2e_1 = -e_1$.

Differensiallashi: $D(e_1) = ae_1,\ D(e_2) = be_1.$

Lokal differensiallashi: $LD(e_1) = ae_1,\ LD(e_2) = be_2.$

Antidifferensiallashi: $AD(e_1) = ae_1,\ AD(e_2) = be_1 - 2ae_2.$

Lokal antidifferensiallashi: $LAD(e_1) = ae_1,\ LAD(e_2) = be_1 + ce_2.$

2) $L(3;1)$: $e_2e_3 = e_1,\ e_3e_2 = -e_1.$

Differensiallashi: $D(e_1) = ae_1,\ D(e_2) = be_1 + de_2 + fe_3,$
$D(e_3) = ce_1 + ee_2 + (a-d)e_3.$

Lokal differensiallashi: $LD(e_1) = ae_1,\ LD(e_2) = be_1 + de_2 + fe_3,$
$LD(e_3) = ce_1 + ee_2 + ge_3.$

Antidifferensiallashi: $AD(e_1) = ae_1,\ AD(e_2) = be_1 + de_2 + fe_3,$
$AD(e_3) = ce_1 + ee_2 + (-a-d)e_3.$

Lokal antidifferensiallashi:
$LAD(e_1) = ae_1,\ LAD(e_2) = be_1 + de_2 + fe_3,$
$LAD(e_3) = ce_1 + ee_2 + ge_3.$

$L(3;2;x)$: $e_1e_3 = e_1,\ e_2e_3 = xe_2,\ e_3e_1 = -e_1,\ e_3e_2 = -xe_2,\ 0 < |x| \le 1.$

Differensiallashi:
$D(e_1) = ae_1,\ D(e_2) = ce_2,\ D(e_3) = be_1 + de_2.$

Lokal differensiallashi:
$LD(e_1) = ae_1,\ LD(e_2) = ce_2,\ LD(e_3) = be_1 + de_2.$

Antidifferensiallashi: $AD(e_1) = ae_1 + ce_2,\ AD(e_2) = be_1 + ae_2,$
$AD(e_3) = -ae_1 - ce_2 - 2ae_3.$

Lokal antidifferensiallashi:
$LAD(e_1) = ae_1 + de_2,\ LAD(e_2) = be_1 + ee_2,$
$LAD(e_3) = ce_1 - de_2 - 2ee_3.$

3) $L(3;3)$: $e_1e_3 = e_1,\ e_2e_3 = e_1 + e_2,\ e_3e_1 = -e_1,\ e_3e_2 = -e_1 - e_2.$

Differensiallashi:
$D(e_1) = ae_1,\ D(e_2) = be_1 + ae_2,\ D(e_3) = ce_1 + de_2.$

Lokal differensiallashi: $LD(e_1) = ae_1,\ LD(e_2) = be_1 + de_2,$
$LD(e_3) = ce_1 + ee_2.$

Antidifferensiallashi: $AD(e_1) = ae_1,\ AD(e_2) = ae_2,$
$AD(e_3) = be_1 + ce_2 - 2ae_3.$

Lokal antidifferensiallashi: $LAD(e_1) = ae_1,\ LAD(e_2) = ae_2,$
$LAD(e_3) = be_1 + ce_2 + de_3.$

4) $L(3;4;x)$: $e_1e_3 = xe_1 - e_2,\ e_2e_3 = e_1 + xe_2,$
$e_3e_1 = -xe_1 + e_2,$
$e_3e_2 = -e_1 - xe_2,\ x \ge 0.$

Differensiallashi: $D(e_1) = ae_1 - be_2,\ D(e_2) = be_1 + ae_2,$
$D(e_3) = ce_1 + de_2.$

Lokal differensiallashi: $LD(e_1) = ae_1 + ee_2,\ LD(e_2) = be_1 + fe_2,$
$LD(e_3) = ce_1 + ge_2.$

Antidifferensiallashi:
$x = 0 \to AD(e_1) = ae_1 + de_2,\ AD(e_2) = be_1 + ae_2,\ AD(e_3) = ce_1 + ee_2 + fe_3.$

$x > 0 \to AD(e_1) = ae_1 - be_2,\ AD(e_2) = be_1 + ae_2,\ AD(e_3) = ce_1 + de_2.$

Lokal antidifferensiallashi:

$x = 0 \to LAD(e_1) = ae_1 + de_2,\ LAD(e_2) = be_1 + ae_2,\ LAD(e_3) = ce_1 + ee_2 + fe_3.$

$x > 0 \to LAD(e_1) = ae_1 - be_2,\ LAD(e_2) = be_1 + ae_2,$

$LAD(e_3) = ce_1 + de_2.$

$L(3;5)$: $e_1e_2 = e_1, e_1e_3 = -2e_2,\ e_2e_1 = -e_1,$

5) $e_2e_3 = e_3,\ e_3e_1 = 2e_2,\ e_3e_2 = -e_3.$

Differensiallashi:

$D(e_1) = ae_1 - 2be_2,\ D(e_2) = be_3,\ D(e_3) = -ae_3.$

Lokal differensiallashi: $LD(e_1) = ae_1 - 2be_2,\ LD(e_2) = be_3,$

$LD(e_3) = -ae_3.$

Antidifferensiallashi: $AD(e_1) = ae_1 + 2de_2 + de_3,$

$AD(e_2) = be_1 - 2ae_2 + ee_3,\ AD(e_3) = ce_1 + 2be_2 + ae_3.$

Lokal antidifferensiallashi: $LAD(e_1) = ae_1 + de_2 + ge_3,$

$LAD(e_2) = be_1 + ee_2 + he_3,\ LAD(e_3) = ce_1 + fe_2 + ie_3.$

$L(3;6)$: $e_1e_2 = e_3,\ e_1e_3 = -2e_2,\ e_2e_1 = -e_3,$

6) $e_2e_3 = e_1,\ e_3e_1 = e_2,\ e_3e_2 = -e_1.$

Differensiallashi: $D(e_1) = -ae_2 - be_3,\ D(e_2) = ae_1 - ce_3,$

$D(e_3) = be_1 + ce_2.$

Lokal differensiallashi:

$LD(e_1) = -ae_2 - be_3,\ LD(e_2) = ae_1 - ce_3,$

$LD(e_3) = be_1 + ce_2.$

Antidifferensiallashi:

$AD(e_1) = ae_1 + ce_2 + de_3,\ AD(e_2) = ce_1 + be_2 + ee_3,\ AD(e_3) = de_1 + ee_2 + (a+b)e_3.$

Lokal antidifferensiallashi: $LAD(e_1) = ae_1 + de_2 + ge_3,$

$LAD(e_2) = be_1 + ee_2 + he_3,\ LAD(e_3) = ce_1 + fe_2 + je_3.$

7) $L(3;-1)$: $e_1e_2 = e_1,\ e_2e_1 = -e_1.$

Differensiallashi: $D(e_2) = ae_1 + de_3,\ D(e_3) = be_1 + ce_2 + ee_3.$

Lokal differensiallashi:

$LD(e_2) = ae_1 + de_3,\ LD(e_3) = be_1 + ce_2 + ee_3.$

Antidifferensiallashi: $AD(e_1) = ae_1,\ AD(e_2) = be_1 - 2ae_2 + ee_3,$

$AD(e_3) = ce_1 + de_2 + fe_3.$

Lokal differensiallashi:

$LAD(e_1) = ae_1,\ LAD(e_2) = be_1 + ee_2 + ge_3,$

$LAD(e_3) = ce_1 + de_2 + fe_3.$

Yuqoridagi natijalarga ko‘ra quyidagi teoremalarga ega bo‘lamiz.

Teorema 1. Ushbu haqiqiy sonlar maydoni ustida aniqlangan kichik o‘lchamli Li algebralarining $L(2,1)$, $L(3,2,x)$, $L(3,5)$, $L(3,6)$, $L(3,-1)$ har qanday lokal differensiallashi differensiallash bo‘ladi. Qolgan $L(3,1)$, $L(3,3)$, $L(3,4,x)$ Li algebralarida differensiallash bo‘lmagan lokal differensiallashlar mavjud.

Teorema 2. Barcha haqiqiy sonlar maydoni ustida aniqlangan kichik o‘lchamli $L(2,1)$, $L(3,2,x)$, $L(3,5)$, $L(3,6)$, $L(3,-1)$, $L(3,1)$, $L(3,3)$, $L(3,4,x)$ Li algebralarida anti-differensiallash bo‘lmagan lokal anti-differensiallashlar mavjud.

FOYDALANILGAN ADABIYOTLAR

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